9/12 Recitation

In this recitation, you will be working with the **Julia5SpecialMatrices** and **Julia6LUFactorization** notebooks.

1. Let's use Julia to help us find an LU decomposition for $A = \begin{bmatrix} 3 & -1 & 0 \\ -9 & 5 & 3 \\ 3 & 5 & 5 \end{bmatrix}$ and then use it to solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$.

   (a) Work through the first section of the **Julia6LUFactorization** notebook to find an LU factorization of A and to solve the system.

   (b) Use the lu(A) function defined in the second section in the Julia notebook to let Julia find an LU factorization of A (it really provides a $PA = LU$ factorization).

   (c) How do the factorizations compare? Are they the same? If they are different, what did Julia do differently? What's going on here?

2. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

   (a) Find a $PA = LU$ factorization by hand.

   (b) Verify your factorization is correct by computing both PA and LU.

3. What row operation does multiplication on the left by E_{ij} encode? What row operation does multiplication on the left by E_{ij}^{-1} encode? Combine these two operations to determine what effect multiplication on the left by $E_{ij}E_{ij}^{-1}$ has. How does this compare to the fact that $E_{ij}E_{ij}^{-1} = I$?

4. Multiplication on the left by a permutation matrix swaps rows. What does multiplication on the right by a permutation matrix do? Use Julia to define some 3×3 permutation matrices and a fixed matrix A. What does multiplication by the permutation matrices on the right do?

5. **Looking Ahead**: Write down a 3×3 lower triangular matrix L and a 3×3 upper triangular matrix U. Use Julia to compute the product $A = LU$ and use Julia to check whether A is invertible by using the inv(A) command. Repeat this with a new pair of lower triangular and upper triangular matrices. And a new pair. And a new pair. Keep repeating until you have found many examples of A being invertible and many examples of A being noninvertible. Try to find a pattern and come up with conditions on the entries of L and U that lead A to be invertible. Once you have a conjecture, prove it!